

Closing Thu: 2.3(part 2)

Closing Tue: 1.5, 4.1

Closing next Thu: 4.2

Entry Task

Solve the system:

$$(i) \quad 4x - y = 3$$

$$(ii) \quad 2x + 3y = 19$$

1.5 & 4.1 Intersections and Graphing

Ch 4 Motivation

We just spent 2 weeks discussing *linear* and *quadratic* **one variable problems**.

We will spend the next week discussing problems with **two variables** (selling two products).

We will learn to maximize and minimize two variable linear functions using the **method of linear programming**.

Before we can do it, we need to know:

1. Find intersections of lines. (1.5)
2. Graph inequalities. (4.1)

An Applied Intersection Problem

(From HW)

Harry borrowed money from the bank and from his life insurance to start a business. The bank loan has a 10% interest rate, and the insurance loan has a 12% interest rate. If the total borrowed was \$100,000 and the total interest in the first year is \$10,700, how much did he borrow from each?

Template for solving:

Step 1: *Label unknowns/collect info*

Let x = “amount borrowed from bank”

y = “amount borrowed from insur.”

amount	x	y
interest		

Step 2: Conditions/Constraints?

Total Borrowed =

Interest in a year =

Step 3: Combine and solve

Example of 4.1 Skills

(a) Graph the inequality: $4x - y \leq 5$

(b) Graph the inequality: $4x + 3y \geq 12$

(c) Shade the points that satisfy ALL four inequalities:

$$4x + 3y \leq 240$$

$$5x - y \leq 110$$

$$x \geq 0$$

$$y \geq 0$$

Section 4.1 / Problem 4: Example of Graphing Inequalities in WebAssign Graphing Tool

Section 4.1 / Problem 4: Graph the region given by the set of inequalities

- (1) $x + 2y \leq 48$; (2) $x + y \leq 30$;
(3) $2x + y \leq 50$; (4) $x \geq 0$; (5) $y \geq 0$

STEP 1: Graph a line for EVERY inequality. In this example you will graph 5 lines. You need to find a couple points on each line that are in the graphing window. We will discuss in lecture how you can quickly find these....

$x+2y = 48$ goes thru (0,24) and (48,0); $x + y = 30$ goes thru (0,30) and (30,0);

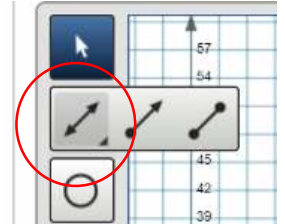
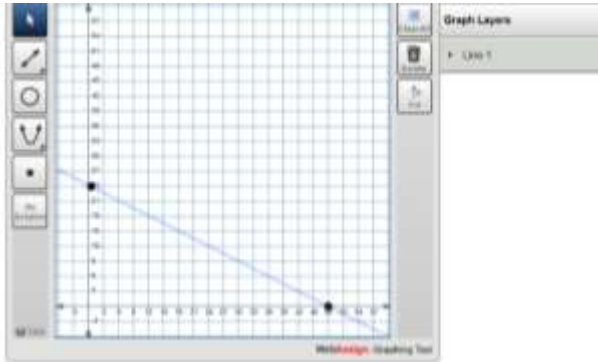
$2x + y = 50$ goes thru (0,50) and (25,0); $x = 0$ goes thru (0,0) and (0,10). $y = 0$ goes thru (0,0) and (5,0).

Plot each line with 3 clicks as follows:

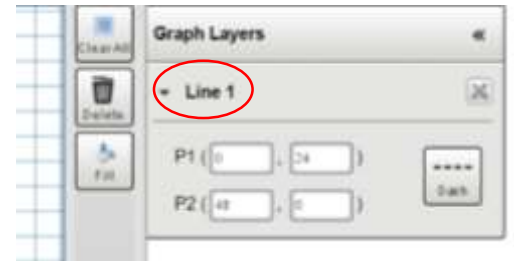
(a) 1 Click - Click on the double-arrow line (do not click "ray" or "line segment").

(b) 2 More Clicks - Click on the two points.

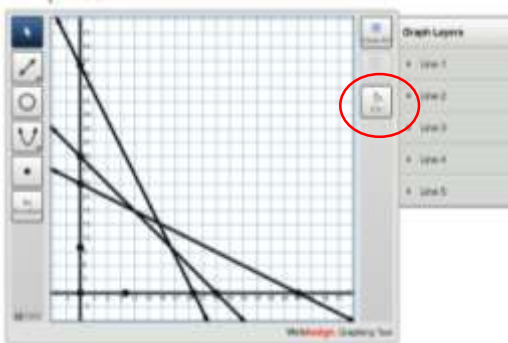
Here I clicked on (0,24) and (48,0) and the line appeared...



NOTE: If you click on the link that says "Line 1" you can see and adjust the points you clicked, it looks like this... (please check this for each line)

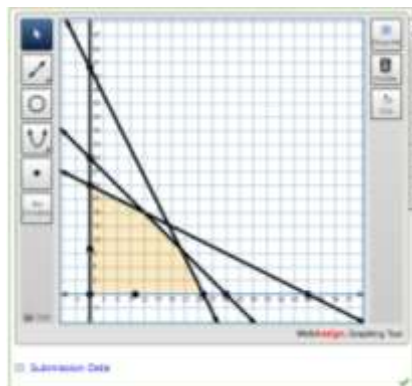


Here is what it looks like after I plotted all 5 lines...



STEP 2: Click "Fill" (shown circled above) and click any point in the region that satisfies all the inequalities.

In this case (1,1) works in all the inequalities you click any point the region containing (1,0). Here is the answer with a "Green Check" for correct, success!



4.2 Linear Programming:

The method of linear programming is a procedure we use to optimize (max/min) an *objective* subject to *constraints*.

Here is the short version:

Step 1: Label the two quantities

Collect information (in a table)

Step 2: Give Constraints and Objective.

Step 3: Sketch the Feasible Region.

Step 4: Find the corners.

Step 5: Plug corners into the Objective.

Biggest output = max

Smallest output = min

4.2: Linear Programming Examples (from the lecture pack)

1. Your company makes two fruit juices. **Apple-Cranberry** is 60% apple juice and 40% cranberry juice. **Cranberry-Apple** is 30% apple and 70% cranberry.

You make \$0.40 profit on each gallon of Apple-Cranberry and \$0.50 profit on each gallon of Cranberry-Apple. Your daily supply is limited to 12,000 gallons of pure apple juice and 11,000 gallons of pure cranberry juice.

Determine **how much of each you should produce daily** to maximize your profit.

STEP 1: “How much of each ... ”

$x =$

$y =$

Gallons =	x	y

STEP 2: Constraints and Objective?

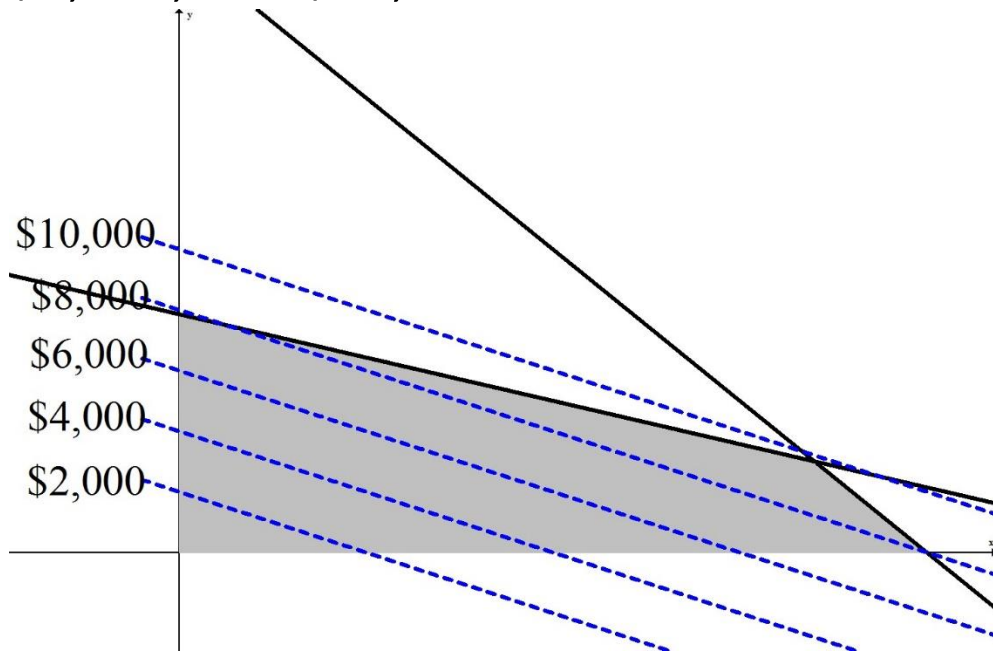
STEP 3: Graph

STEP 4: Corners

STEP 5: Evaluate objective

The feasible region for the juice example

Dotted lines corresponding to all points where profit is \$2,000, \$4,000, \$6,000, \$8,000, and \$10,000 are drawn.



2. Gina inherits a large sum of money and a bunch of pet cages from an aunt.

She decides to rescue some unwanted pets from a shelter.

She has 20 cages that can each house either a bunny or a ferret.

On average it costs \$0.60 a day to feed one ferret and \$0.80 a day to feed one bunny. Gina can budget no more than \$14.40 a day for pet food. But cuddliness is an issue for Gina. She figures that bunnies are twice as cuddly as ferrets. That is, ferrets are each worth one cuddle-unit, while bunnies are each worth two.

Determine **how many of each pet** Gina should adopt to maximize cuddliness while staying within her budget and without buying more cages.

STEP 1: "...how many of each pet..."

$x =$

$y =$

Total pets =	x	y

STEP 2: Constraints and Objective?

STEP 3: Graph

STEP 4: Corners

STEP 5: Evaluate objective

(,) \rightarrow

(,) \rightarrow

(,) \rightarrow

(,) \rightarrow

3. (From Homework) Two foods contain only proteins, carbohydrates, and fats.

Food A costs \$1 per pound and contains 30% protein and 50% carbohydrates.

Food B costs \$1.50 per pound and contains 20% protein and 75% carbohydrates.

What **combination of these two foods** provides at least 1 pound of protein, 2.5 pounds of carbohydrates, and 0.25 pounds of fat at the lowest cost?

STEP 1: "What ... these two foods..."

x =

y =

amount =	x	y

STEP 2: Constraints and Objective?

STEP 3: Graph

STEP 4: Corners

STEP 5: Evaluate objective

(,) →

(,) →

(,) →

4. Your company makes two household cleaners: Miracle Bathtub Cleaner and Speedex Floor Cleaner. Your daily production of both cleaners combined is limited to 2,000 gallons. Your daily sales of Miracle Bathtub Cleaner never exceed 1,200 gallons, and your daily sales of Speedex Floor Cleaner never exceed 1,400 gallons. Finally, you make \$1.00 profit on each gallon of Miracle Bathtub Cleaner that you sell and \$2.00 on each gallon of Speedex Floor Cleaner that you sell.

Determine **the amount of each cleaner you should produce** in order to maximize profit.

STEP 1: "...amount of each cleaner..."

$x =$

$y =$

	x	y

STEP 2: Constraints and Objective?

STEP 3: Graph

STEP 4: Corners

STEP 5: Evaluate objective

(,) \rightarrow

(,) \rightarrow

(,) \rightarrow

(,) \rightarrow

(,) \rightarrow